Hyperbolic distortion and boundary behaviour of holomorphic functions on the unit disk

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Denote $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and let $f : \mathbb{D} \to \mathbb{D}$ be a holomorphic function. The *hyperbolic distortion* $D_h f$ of f is defined by

$$D_h f(z) := \frac{(1 - |z|^2)|f'(z)|}{1 - |f(z)|^2} \qquad \text{for } z \in \mathbb{D}.$$

The classical Schwarz-Pick lemma states that $D_h f(z) \leq 1$ for all $z \in \mathbb{D}$ with equality for one — and hence every — $z \in \mathbb{D}$ if and only if f is a holomorphic automorphism of $\mathbb{D}$. Therefore, we can think of $D_h f$ as an indicator of how "close" f is to being an automorphism. In this sense the equality case in Schwarz-Pick lemma, that is the condition $D_h f(z) = 1$ for $z \in \mathbb{D}$, is a characterization of a specific property of the function f, that is f being an automorphisms of $\mathbb{D}$, in terms of $D_h f$.

This talk deals with boundary versions of the case of equality in Schwarz-Pick lemma: we discuss several conditions on the asymptotic behaviour of $D_h f$ at a boundary point $\sigma \in \partial\mathbb{D}$ that characterize different properties of the boundary behaviour of the function f (at σ).