Boundedness, compactness and Schatten class for Rhaly matrices

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In this talk, we discuss the *Rhaly operator* R_α, that acts on ℓ^2 as the infinite matrix

$$R_\alpha = \begin{pmatrix} \alpha_0 & 0 & 0 & 0 & 0 & \cdots \\ \alpha_1 & \alpha_1 & 0 & 0 & 0 & \cdots \\ \alpha_2 & \alpha_2 & \alpha_2 & 0 & 0 & \cdots \\ \alpha_3 & \alpha_3 & \alpha_3 & \alpha_3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Rhaly matrices arise as the natural generalization of the classical Cesàro operator: choosing

$$\alpha_k = \frac{1}{k+1}, \qquad k \in \mathbb{N},$$

then R_α coincides with the classical Cesàro operator $\mathcal{C}$, that assigns to each $f \in \ell^2$ the averages

$$\mathcal{C}f(k) = \frac{1}{k+1} \sum_{j=0}^{k} f(j), \qquad k \in \mathbb{N}.$$

We provide new characterizations of the boundedness and compactness of R_α on ℓ^2, and we completely characterize its membership in the p-Schatten class $\mathcal{S}^p(\ell^2)$, for $1 < p < \infty$. This talk is based on a joint work with Carlo Bellavita and Giorgos Stylogiannis, [1].

References

[1]   C. Bellavita, E. Dellepiane, G. Stylogiannis, Boundedness, compactness and Schatten class for Rhaly matrices, *Journal of the London Mathematical Society*, Vol. 112, no.4 e70304 (2025). https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/jlms.70304